Deformation of framed structures due to tunnelling induced movements – a rational and practical assessment approach

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It is common practice to assess the deformation of structures due to tunnelling induced effects by assuming that foundation movements coincide with greenfield tunnelling movements. This often overestimates the resulting movements and stresses on the structures and may fail to give the correct deformed shape for detailed damage assessment. This paper proposes a novel approach to study the interaction between tunnels and framed structures, based on the translational and rotational stiffnesses of foundations from the field of seismic engineering. This approach can readily be deployed in routine civil engineering design and tunnelling impact assessment.

This paper contains 9 pages and 3237 words.

INTRODUCTION

The calculation of the deformation of buildings in response to tunnelling induced ground movements is a regular undertaking for geotechnical engineers tasked with carrying out impact assessments for tunnelling projects in an urban environment. The author has had the opportunity to approach this problem from the perspectives of tunnel designer, client's representative and regulatory authority.

To consider soil-structure interaction properly normally involves the use of finite element analysis. This requires an appropriate and realistic soil constitutive model and consideration of superstructure stiffness. However, superstructure details are not easily modelled in common commercial 2D geotechnical softwares without gross simplification. The need to assess a large number of structures, each with different geometry and proximity to the tunnel, means that detailed modelling for each structure is often not practical.

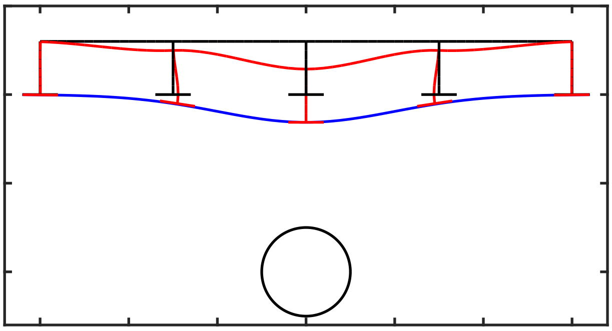

Figure 1. Displacement of a framed structure (red) following greenfield settlement profile (blue)

Given these difficulties, it is common practice to assume that surface structures deform to the same extent as a greenfield displacement profile (Figure 1), an assumption which is highly conservative (Farrell and Mair, 2011) because this assumes that the structures are infinitely flexible (Franzius et al., 2006). In practice, further conservatism results when engineers impose such deformations, which could only occur in infinitely flexible structures, onto buildings with finite stiffness to calculate internal forces. It is to be expected that the resulting bending moments and forces could be greatly out of proportion.

Methods such as the relative stiffness approach (Potts and Addenbrooke, 1997) take into account building stiffness when predicting building deformation in response to tunnelling induced movements. However, they are not easily generalised for application in more complex structural shapes such as framed or arch structures.

While researching into this topic, the author was introduced to research findings on the stiffness of shallow foundations in the field of dynamic analysis of soil-foundation-structure systems under seismic loading. It became apparent that the same stiffnesses can also be applied as springs to analyse the response of foundations to static loadings such as tunnelling induced movements.

In this paper, a new method is proposed for calculating the response of simple framed structures to tunnelling induced movements, using a foundation stiffness approach which is efficient and suitable for routine application in tunnelling impact assessments.

PROPOSED METHODOLOGY

The proposed approach is illustrated with a plane frame shown in Figure 2. Beams and columns are defined by their lengths, flexural and axial rigidities. Footings are not directly modelled as structural members, but are instead represented by foundation springs. The proposed methodology involves the following steps:

(a) Establishing ground movements under greenfield conditions

(b) Evaluating foundation stiffness based on foundation geometry and soil modulus

(c) With the foundation stiffness, calculating the set of forces and moments equivalent to greenfield movements

(d) Solving for displacements of foundation and framed structure under the set of tunnelling forces and moments

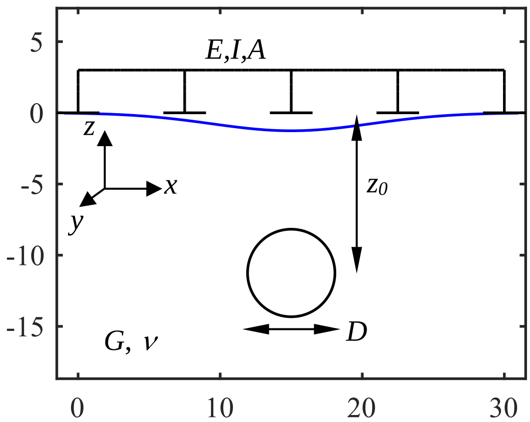

Figure 2. Plane frame on rigid footings founded on elastic half-space

Greenfield tunnelling displacement profile

A greenfield tunnelling displacement profile $\{u_{gf}\}$ is readily available from past experience or field measurements at reference sites under similar tunnelling and ground conditions.

While other methods such as finite element analysis may give an estimate of the greenfield displacement profile, New and O'Reilly (1991) cautions that the sensitivity of these methods to input parameters has meant that it may be less reliable than empirical predictions. This remains true in practice.

In clays and under the condition of constant volume, it is assumed that the profile of ground surface settlement can be described by a Gaussian curve in the following form (Grant and Taylor, 2000):

$$u_z = S_{max} \exp\left(-x^2 / 2i^2\right) \quad (1)$$

$$u_x = u_z\, x / 1.538 z_0 \quad (2)$$

$$S_{max} = V_L / \sqrt{2\pi}\, i. \quad (3)$$

The distance i between the tunnel centre line and the point of inflexion of the settlement trough may be defined relative to the tunnel depth by a trough width parameter K as

$$i = K z_0 \quad (4)$$

, with $K = 0.5$ being a reasonable value (Mair et al., 1993).

In sands, a modified Gaussian curve allowing for a narrower profile may be more suitable with a shape function parameter n (Vorster et al., 2005):

$$u_z = \frac{n}{(n-1) + \exp\left[\alpha\,(x/i)^2\right]}\, S_{max} \quad (5)$$

$$n = e^{\alpha} \frac{2\alpha - 1}{2\alpha + 1} + 1. \quad (6)$$

The standard Gaussian shape corresponds to $n = 1$.

The rotation under greenfield conditions is taken as the first derivative of the vertical greenfield settlement curve:

$$u_r = \frac{du_z}{dx} \quad (7)$$

The above displacements are taken as the greenfield movements that would occur at the centroid of the footings being considered. This is an approximation because the distribution of greenfield displacements varies along the width of the actual footings. The error is not expected to be large unless the footing being considered is sizable compared to the extent of the greenfield profile.

The standard Gaussian profile is assumed in this paper, except in the Validation section below.

The actual displacement profile will be modified by the presence of the superstructure and its foundations.

Stiffness of foundations on elastic half-space

Gazetas (1991) proposes the static stiffnesses of a shallow rigid foundation of an arbitrary shape on the surface of an elastic half-space. The foundation considered has six degrees of freedom and the stiffness for each of the degrees of freedom is given in Table 1. Numerical and physical modelling of foundations formed the basis of the derivation. The simplicity of the formulation and the small number of parameters make it readily usable by practising engineers with minimal computational requirements. It also enables the user to develop a feel for the relative importance of the parameters and exercise engineering judgement accordingly (Dobry, 2014).

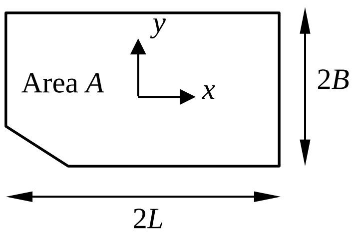

Figure 3. Definition of dimensions and reference axes for footing of arbitrary shape after Gazetas (1991), x and y axes being the minor and major principal axes respectively for the circumscribed rectangle

Table 1. Static spring stiffnesses after Gazetas (1991)

Mode	Stiffness (force per unit displacement)
Vertical (z)	$K_z = [2GL/(1-\nu)](0.73 + 1.5\,\chi^{0.75})$ with $\chi = A/4L^2$
Horizontal (y)	$K_y = [2GL/(2-\nu)](2 + 2.5\,\chi^{0.85})$
Horizontal (x)	$K_x = K_y - GL\,[0.2/(0.75-\nu)](1 - B/L)$
Rotational (about x)	$K_{rx} = [G/(1-\nu)]\,I_x^{0.75}\,(L/B)^{0.25}\,(2.4 + 0.5B/L)$
Rotational (about y)	$K_{ry} = [3G/(1-\nu)]\,I_y^{0.75}\,(L/B)^{0.15}$
Torsional (about z)	$K_{rz} = 3.5G\,I_z^{0.75}\,(B/L)^{0.4}\,(I_z/B^4)^{0.2}$

In a plane frame problem (x-z plane), the relevant foundation stiffness matrix is

$$[K_f] = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_z & 0 \\ 0 & 0 & K_{ry} \end{bmatrix} \tag{8}$$

, which is a function of the foundation geometry, shear modulus and Poisson's ratio.

Method of analysis

The behaviour of the structural frame is described by

$$[K]\{u\} = \{F\} \tag{9}$$

where $[K]$ is the stiffness matrix of the system, $\{u\}$ is the displacement of the frame and force vector $\{F\}$ represents the loading on the structure from the tunnelling induced movements.

The stiffness of the soil-structure system is

$$[K] = [K_s] + [K_f] \tag{10}$$

where $[K_s]$ is the stiffness of the frame consisting of discrete elements modelled as Euler-Bernoulli beams.

The greenfield tunnelling profile gives rise to an equivalent set of forces on the structure given by

$$\{F\} = [K_f]\,\{u_{gf}\}. \tag{11}$$

Hence the displacement of the structure $\{u\}$ can be obtained by solving Equation (9) directly. The following are assumed under this approach:

(a) The soil response to loading at the foundation level is not affected by the presence of the tunnel.

(b) The founding soil is homogeneous and elastic.

(c) The footings are rigid.

(d) The footings remain in contact with the founding soil.

(e) The interaction between individual footings is negligible.

By modelling the tunnelling induced effects as equivalent forces on the system instead of imposing the greenfield displacement to the footings directly, the contribution of the stiffnesses of the superstructure and foundation towards the final structural displacements is taken into account.

Superimposing greenfield displacements directly to the structure implies $[K_s] = 0$, i.e. an infinitely flexible superstructure.

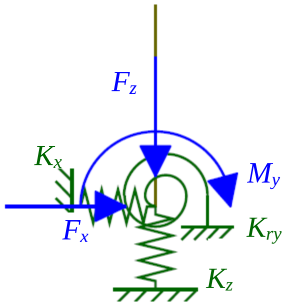

Figure 4. A column connected to a rigid footing represented by foundation springs [K_f] (green) and subjected to equivalent tunnelling forces {F} (blue)

In this paper, the direct stiffness method with 0.5 m beam elements is employed with a custom finite element code. The same process can be adopted using routine commercial structural softwares by applying translational and rotational springs to model the foundation. An example is shown in Figure 4.

MODELLING RESULTS

A 30 m wide, four-span single-storey frame located centrally above a tunnel is analysed for illustration. Input parameters are summarised in Table 2.

The displacement of the frame is presented in Figure 6 and Table 3. Results indicate that the central footing settles by an amount equal to 91% of the greenfield settlement. The side footings, on the other hand, settle slightly more than under greenfield conditions. It is therefore evident that the stiffness of the framed structure as a whole affects significantly the distribution of deformation within the frame. It is clear that the greenfield profile does not always give the upperbound values for structural movements.

The horizontal movements and rotations of side footings are only approximately 10% of the corresponding greenfield values, as a result of the flexural and axial rigidity of the columns and beams.

Table 2. Input parameters

Parameter	Input value
Tunnel diameter D	5 m
Tunnel depth z_0	10 m
Column spacing	7.5 m
Column height	3 m
Beam dimensions	1000 mm × 350 mm depth
Column dimensions	1000 mm (along y) × 350 mm (along x)
Modulus of frame	70 MPa
Footing dimensions	3 m × 3 m
Modulus of soil	25 MPa
Poisson's ratio of soil	0.25

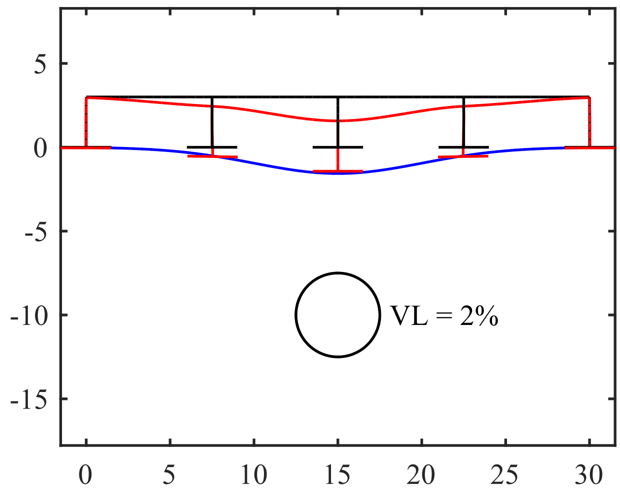

Figure 6. Deformed shape (displacements ×50 times)

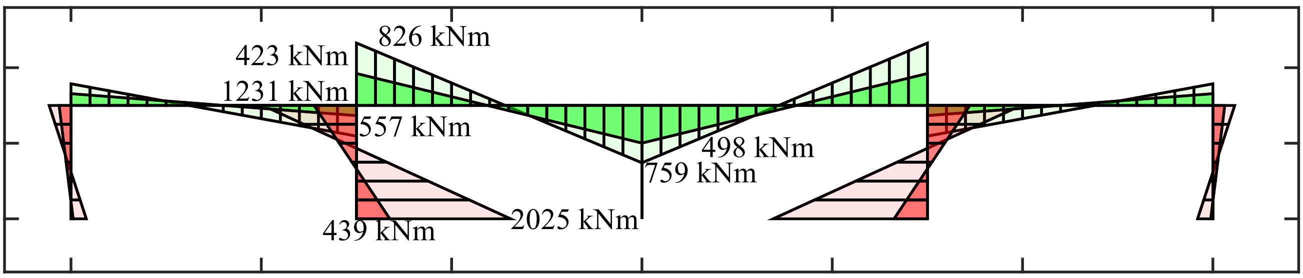

Figure 5. Bending moments for deformed frame (i) under imposed greenfield movements (light colours) M_{gf} and (ii) considering soil-structure interaction (dark colours) M_{ss}

As a consequence of the reduced movements, the maximum bending moments in the beams and columns are 66% and 22% of the values in a fixed frame with greenfield movements imposed (Figure 5). The greatest reduction in bending moment occurs at the base of the columns, where it is most sensitive to horizontal displacements at the foundations.

Table 3. Summary of foundation movements (footings numbered from left to right)

Displacements	Greenfield	Modelling
Footing 1 (Footing 5 opposite in signs for u_x, r_y)		
u_z	-0.35 mm	-0.77 mm
u_x	0.34 mm	-0.46 mm
r_y^*	-0.0002	-0.0000
Footing 2 (Footing 4 opposite in signs for u_x, r_y)		
u_z	-10.16 mm	-11.09 mm
u_x	4.95 mm	0.65 mm
r_y^*	-0.0030	-0.0003
Footing 3		
u_z	-31.30 mm	-28.60 mm
u_x	0 mm	0 mm
r_y	0	0

* +ve anti-clockwise

VALIDATION

In this section, results of the proposed method are validated against published results using other more rigorous methods of analysis.

Reference is made to results by Franza and DeJong (2017) who derived the foundation stiffness matrices of framed structures on footings founded on an elastic continuum. The basis of their formulation is the integral forms of Mindlin's solutions given by Vaziri et al. (1982). Their foundation stiffness differs from Equation (8) in that interactive springs are used, while the Gazetas springs used under this paper are uncoupled. Franza and DeJong (2017) also compared their approach with numerical results by Giardina et al. (2015) on the basis of centrifuge tests by Farrell et al. (2014), with good agreement.

Under this section, the same modified greenfield displacement profile used by Franza and DeJong (2017) is adopted instead of the standard Gaussian distribution. The vertical greenfield settlement profile used corresponds to n = 0.1184.

Figure 7 shows that the movements of the five footings determined from Franza and DeJong (2017)'s elastic continuum solution are reproduced using Gazetas (1991)'s spring stiffnesses as outlined in this paper. The calculated displacements following the approach under this paper, in particular the rotations of the footings, are consistent with the elastic continuum solution based on Mindlin's equations. It is encouraging to observe that the simplified method under this paper produces consistent results.

The fact that the rotations of the side footings are in opposite directions to the gradient of the greenfield profile is explored in the following section.

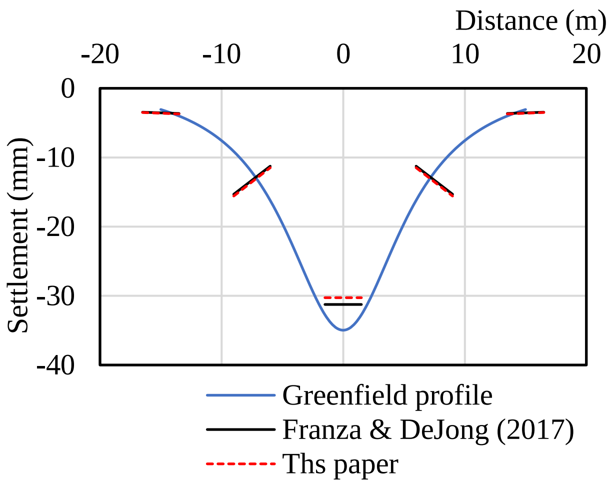

Figure 7. Comparison of foundation movements with other published results

DISCUSSION

Rational approach

The proposed approach makes no assumptions as to the deformation of the structure. The calculated settlements and rotations may be greater or smaller than under greenfield conditions, depending on the layout of the structure and the tunnel position. Parts of the structure may be forced to settle or rotate more than the greenfield profile due to the stiffness of the frame. This effect cannot be assessed if the structure is assumed to follow the greenfield profile. It is shown that this is not always on the conservative side.

In particular, certain combinations of structure stiffness and foundation stiffness may result in deformations which is radically different from greenfield conditions. For example, Figure 8 shows the same tunnel-frame system, with the foundation dimensions changed from 3 m × 3 m to 2 m × 2 m. Smaller footings, hence smaller translational and rotational foundation stiffnesses, result in a rotation of the footing which is in opposite direction to the gradient of the greenfield profile, as the superstructure stiffness becomes more dominant.

In extreme cases, the tension side of a column could occur at the opposite side of the member. The ability of a method of analysis to arrive at a realistic deformed shape has potentially huge implications on the design of mitigation or strengthening works and on the details of structural monitoring schemes.

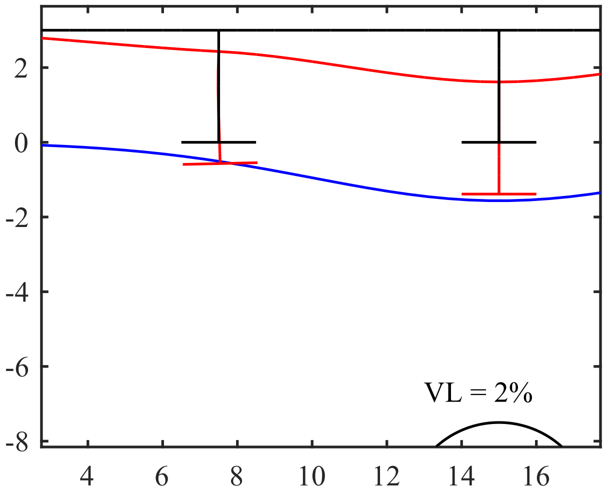

Figure 8. Deformed shape showing footing rotation in opposite direction to greenfield profile (displacements ×50 times)

Effect of soil elastic modulus

The foundation stiffness used in this paper assumes a homogeneous elastic half-space with a constant elastic modulus. In its formulation for seismic engineering purposes, Gazetas (1991) considered that the soil layer does not need to be very thick for this assumption to be valid because the horizontal and rotational oscillations produce only very shallow dynamic pressure bulbs.

Contrary to other common geotechnical design processes, a lower soil elastic modulus for the present analysis is not more critical. Figure 9 compares the maximum bending moment at critical locations of the frame with corresponding values in a fixed frame with greenfield movements imposed. A bending moment ratio *MR* of 1 corresponds to the case of a fixed frame. Given the same volume loss and greenfield movements, a stiffer soil exerts greater forces onto the frame and therefore results in greater movements and bending moments. Hence the “moderately conservative” or “characteristic” values of elastic modulus required under geotechnical designs should be construed accordingly by taking an appropriately high value for soil elastic modulus.

As a corollary to this positive relationship between soil modulus and structural movements, the soil would have to be very stiff to overcome the stiffness of the superstructure and to force the foundation to displace according to the greenfield profile. For the example problem in Table 2, the soil elastic modulus would have to be more than 100 times greater, i.e. in the order of 2.5 GPa, to cause the structure to behave like a fixed frame subjected to greenfield movements (Figure 1). A soil stiffness of this order is not normally expected to occur near the ground surface. Even if such a soil material at foundation level is possible, it is questionable whether a large volume loss is feasible in such stiff material.

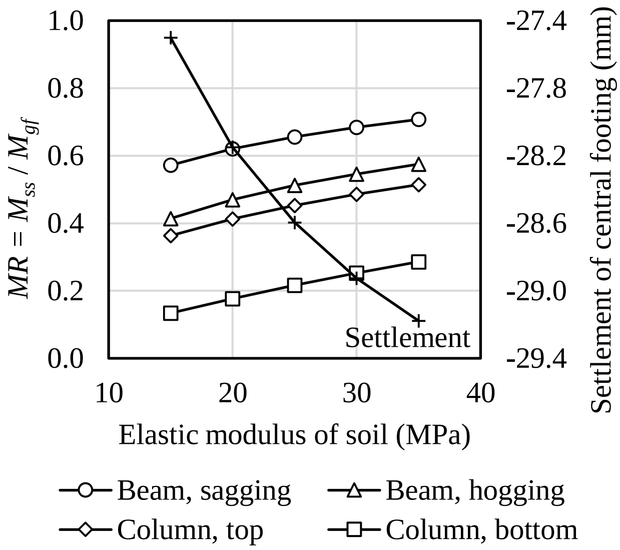

Figure 9. Effect of elastic modulus of founding soil on bending moment and settlement

In addition, a fixed frame with greenfield movements assumes that no slippage occurs between the foundations and the soil. In typical footings with no additional shear connections with the founding soil, the validity of this assumption is questionable for the case of stiff frames subjected to large tunnelling movements.

This is therefore a further indication that a framed structure should not normally be assumed to follow a greenfield settlement profile.

Practical application in tunnelling industry

Due to the simple formulation of the matrices, the proposed approach is an efficient method to consider soil-structure interaction. The foundation stiffnesses can be assigned to the footing nodes in common structural numerical programmes to obtain the deformed shapes directly. Because the Gazetas stiffnesses cover all six degrees of freedom, the analysis can readily be expanded to analyse 3D frames and different orientations of the tunnel. A useful tunnelling application would be the study of structural response to 3D ground settlement trough caused by an advancing tunnel.

In conventional plane strain geotechnical finite element programmes, structural members have to be modelled as infinitely long plate elements. It is necessary that an iterative process of conversion and validation be performed such that 3D effects are properly accounted for in a 2D analysis. No such conversion is necessary in this paper.

Given the straightforward approach in modelling tunnelling impact as equivalent forces, the proposed approach allows analyses of different structural forms or tunnelling locations to be conducted efficiently. Different positions or orientations of the tunnel including those oblique to the plane of the frame could be readily analysed by resolving the component of the equivalent forces along and perpendicular to the plane of the frame. Without the need to simplify superstructure and foundation elements into plane strain elements for 2D analysis, structural details can be added and studied efficiently (e.g. Figure 10). With appropriate formulation of the stiffness matrices, it is possible to model the effects of bracings, facades or infilled panels as well.

This proposed approach may assist designers, clients and regulators to determine the tunnelling impact in a rational manner. It has been the author's experience that excessive conservatism may not always lead to a safe solution. Extensive mitigation or stabilisation works deemed necessary following an unduly conservative impact assessment could themselves generate new sources of adverse impact and workmanship issues.

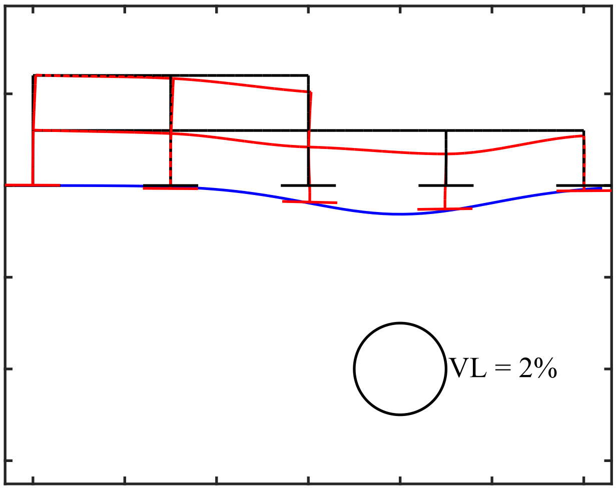

Figure 10. Deformation of two-storey framed structure

FUTURE WORK

This paper is prepared in the course of the author's research on the tunnelling response of structures.

The behaviour of more massive and embedded foundations such as bridge piers is not at present well understood. It is intended that a Mindlin based solution be developed for an embedded foundation on elastic half-space. Further research would aid the study into the tunnelling response of embedded foundations such as basements and bridge piers.

Despite more rigorous methods of analysis, simplified methods would continue to have their place in routine design and in the validation of new approaches.

CONCLUSION

This paper demonstrates that it is feasible to assess the deformation of framed buildings due to tunnelling induced movements by incorporating foundation stiffness springs at the footings.

It has been shown that framed structures should not realistically be assumed to following greenfield movements, in which case the true behaviour of the structure may not be captured. Unlike flexible raft foundations or ground beams which may, albeit conservatively, be assumed to follow greenfield movements, adopting the same approach for framed structures on separate footings may not always be on the safe side.

A rational method for considering soil-structure interaction has been presented. This is amenable to routine engineering analysis using commercial computer programmes.

ACKNOWLEDGEMENTS

The author wishes to thank Prof Giulia Viggiani of the University of Cambridge for her comments on the manuscript. Dr Andrea Franza of the Universidad Politécnica de Madrid introduced the author to the work of Prof George Gazetas, the application of whose work forms the basis of this paper. Prof Sinan Acikgoz of the University of Oxford also provided valuable advice on the numerical modelling approach.

The author is also grateful to the Geotechnical Engineering Office under the Civil Engineering and Development Department of the Government of Hong Kong S.A.R. for sponsoring the author's studies.

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